DOCUMENT RESUME

ED 350 166 SE 053 226

TITLE	New Jersey's Core Course Proficiencies: Mathematics.
INSTITUTION	New Jersey State Dept. of Education, Trenton. Div. of General Academic Education.
PUB DATE	May 90
NOTE	86p.
AVAILABLE FROM	New Jersey State Dept. of Education, 225 West State Street, CN 500, Trenton, NJ 08625.
PUB TYPE	Legal/Legislative/Regulatory Materials (090) -- Guides - Non-Classroom Use (055)
EDRS PRICE	MF01/PC04 Plus Postage.
DESCRIPTORS	*Algebra; *Core Curriculum; Equations (Mathematics); *Geometry; High Schools; *Mathematics Curriculum; Mathematics Education; Measurement; Number Systems; Problem Solving; *Secondary School Mathematics; *State Norms; Symbols (Mathematics); Thinking Skills
IDENTIFIERS	*New Jersey

ABSTRACT

This document presents Core Course Proficiencies for the State of New Jersey whose purpose is to ensure that all students have equal access to the fundamental knowledge and skills critical to achieving success in mathematics. After an introduction providing rationale and background, the Core Course Proficiencies are presented in three sections. The algebra section is divided into two parts for Algebra I and Algebra II. The Algebra I and Algebra II proficiencies are each presented in five clusters: equations and inequalities; number systems; the language of algebra; algebraic expressions; and relations, functions, and graphing. Problem solving is woven throughout the clusters, rather than identified as a cluster within itself. Nine core proficiencies for the Geometry section are presented: relationships of points, lines, planes, and space; systems of measurement; reasoning; coordinate geometry; congruency; properties of polygons and circles; transformations; similarity; and symbolism and notation. The general themes of problem solving, higher-order thinking, communication, connections, using concrete models and manipulatives, estimation, and using calculators and computers are reflected throughout these proficiencies. Six core proficiencies are presented for General Math I, II, and III: numerical operations; patterns and functions; data analysis; measurement and geometry; fundamentals of algebra; and problem solving. (MDH)

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NEW JERSEY'S

CORE COURSE PROFICIENCIES:

Mathematics

Saul Cooperman
Commissioner

Joel Bloom
Assistant Commissioner
Division of General Academic Education

New Jersey State Department of Education
225 West State Street
CN 500
Trenton, New Jersey 08625

SE 053226

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FOREWORD

New Jersey public education is increasingly dedicated to ensuring that all students have access to a quality education, one that will prepare them to be effective and productive citizens. To ensure that quality and students' access to the essential knowledge and skills, regardless of where our young people reside, educators from all parts of the state have worked collaboratively to identify core course proficiencies in those mathematics and science courses most students take to meet the high school graduation requirements.

The Core Course Proficiencies initiative is an outgrowth of the recommendations from two study panels convened during the past five years: one to review and revise high school graduation requirements, and the other to consider the need for and impact of establishing a common core of knowledge and skills in specified high school courses. Both panels identified the need for a common core in the content areas, both identified a need to upgrade the high school curriculum, and both stressed concern for assuring equity in education for all New Jersey students. The Core Course Proficiencies initiative meets these three concerns head-on.

Included within the initiative is an exhaustive five-year plan to convene panels of outstanding educators in each of the specified courses in order to identify proficiencies in their respective subject areas. The results of their efforts will be shared with educators statewide in order to obtain feedback that will help shape the proficiencies to be recommended for adoption by the State Board of Education.

With the efforts of the first two panels and input from local educators across the state, we now have core course proficiencies in six science and six mathematics courses. These proficiencies are identified in two documents that will be distributed to local districts in October, 1990. The proficiencies are but a first step toward building a stronger curriculum and ensuring equity for all students.

Local boards of education are required to incorporate these core course proficiencies into the curricula. All students can learn these core course proficiencies. It is up to New Jersey educators to provide learning opportunities that will enable them to do so.

Saul Cooperman
Commissioner

CORE COURSE PROFICIENCIES
Mathematics

David Glatzer, Co-chair	West Orange Public Schools
Charles Kuzminski, Co-chair	Howell Township Public Schools

Algebra I, II Panels

James Benson	Fairleigh Dickinson University
Carol Canfield	West Milford Public Schools
Gail Chaskes	Gloucester County Vocational Schools
Patricia Curtis	Ridgefield Park Public Schools
Frank Gardella	East Brunswick Public Schools
Richard Giantisco	Bethlehem Township Public Schools
Diana Harmon	East Orange Public Schools
Jerome Harris	Trenton High School
Lewis Hirsch	Rutgers University
Delores Janns	Spotswood High School
Cheryl Richardson	Jersey City Public Schools
Dennis Watson	Clearview Regional High School

Geometry Panel

Janet Caldwell	Glassboro State College
Sam Carabases	Camden City Public Schools
Iris Elliott	Rancocas Valley Regional High School
James Rahn	Southern Regional High School
Lawrence Silverman	Boonton High School
Thomas Smith	Hunterdon Central High School
Frank Volpe	Bernards Township Public Schools

General Mathematics I, II, III Panels

Marcy Lehigh	Secaucus Public Schools
Martin Matula	Delaware Valley Regional High School
June Ramondetta	Washington Township Public Schools
Ellen Brockman	Totowa Public Schools
Thomas Buscemi	Vineland High School - South
Regina Cullen	West Essex Regional High School
Elaine Danelli	Evesham Township Public Schools
Joseph Kuhn	Price Waterhouse
Audrey Langan	Bayonne High School
Edward Luick	Lakewood Middle School
Barbara Williams	Elizabeth Public Schools

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NEW JERSEY'S CORE COURSE PROFICIENCIES

Background

In May, 1980, a statewide committee studying high school graduation requirements recommended that the establishment of student proficiencies for each course and the continuous upgrading of proficiencies be studied. Responding to that recommendation, a Statewide Panel on High School Proficiencies, appointed by Commissioner of Education Saul Cooperman, studied the issues of proficiencies: how they should be developed and assessed and what their impact would be. In light of the panel's findings, which were reported in December, 1987, the Commissioner presented recommendations to the State Board of Education in June, 1988. These recommendations included establishing core proficiencies statewide for those courses most students take to meet the State Board of Education curriculum requirements for graduation in New Jersey. The proficiencies were to be identified by panels of outstanding professionals in each of the identified content areas through curriculum convocations and school-district review. The Core Course Proficiencies initiative was adopted by the State Board of Education at its May, 1989, meeting.

Purpose

The purpose of the Core Course Proficiencies initiative is to ensure that all students, regardless of where they reside, have equal access to the fundamental knowledge and skills critical to achieving success. From district to district, New Jersey students must meet the same course and credit requirements in: English; mathematics; science; social studies; fine, practical, and/or performing arts; career education; and physical, health, and safety education. Until the State Board of Education established the initiative in May, 1989, no consideration had been given to assure that students taking similar courses for high school graduation were acquiring the same essential core knowledge and skills.

Although the state sets the general requirements for graduation, determining the skills and knowledge that constitute proficiency in specific subjects has been left to individual districts; and standards vary widely. Such discrepancies can cheat students and undermine New Jersey's efforts to guarantee the quality of its diploma.

The expected outcomes of the New Jersey Core Course Proficiencies initiative are:

- The identification of the essential knowledge and skills that students should master in those courses most students take to meet the high school graduation requirements.

- The upgrading of the high school curriculum across the state.

Feedback during all phases of the initiative has been extremely positive. Most participants in the eighteen-month identification process have indicated that the development of core course proficiencies is critical for improving curriculum and "the right thing to do." At the heart of this effort has been

and continues to be the involvement of dedicated educators, including content area teachers, administrators, and supervisors, as well as representatives from business and industry and from higher education.

Process

In July, 1989, panels of outstanding educators in mathematics and science (approximately 35 in each) met to identify core course proficiencies for six mathematics courses (Algebra I and II, Geometry, and General Mathematics I, II, and III), and six science courses (Biology, Chemistry, Physics, Earth Science, General Science, and Physical Science). Panel members included content area teachers, administrators and supervisors, as well as representatives from business and industry, and higher education.

During August, educators statewide were invited to attend two mathematics and science convocations in order to review panel drafts of the proficiencies. The panels subsequently reconvened on October 16 to revise the proficiencies based on the input of the 350 educators who attended the August convocations. Approximately 100 high school districts were then selected to review the mathematics and science proficiencies and to provide feedback on them. This feedback served as the basis for final revisions of the mathematics and science proficiencies, which are being presented to the State Board of Education for approval. School districts will receive the approved proficiencies in October, 1990.

MATHEMATICS COURSE CORE PROFICIENCIES

INTRODUCTION

As we speed to a new millennium and a world of changing priorities, it should not come as a surprise that we in education find ourselves challenging many of the assumptions that have formed the cornerstones of our profession. Some we will continue to support. However, many of the ideas that helped prepare America so well for the industrial age are simply not appropriate in a changing society propelled by information and technological advancements.

Mathematics provides classic examples of the need to change. Technology has dramatically altered both the nature and application of mathematics in today's world. In response, we have debated for years issues such as the use of calculators in mathematics classrooms. Only recently have we begun to nurture the changes thus shaping their impact on the human experience.

It is in this spirit that the core course proficiencies that follow were developed. Understanding that its efforts would provide a blueprint for change in New Jersey mathematics classrooms, the committee looked to national and state research for direction. It relied heavily on national reports such as NCTM's Curriculum and Evaluation Standards for School Mathematics and Project 2061 from the American Association for the Advancement of Science, both of which are attempts to provide guidelines for reform in school mathematics. Committee members also listened carefully to their colleagues statewide before finalizing their recommendations. The resulting document is truly a collaborative vision of New Jersey high school mathematics courses in the decade of the 90's.

The changes are not restricted to content. We feel strongly that the teaching of mathematics must also change. The paper-and-pencil exercises which have dominated mathematics instruction must make room for exercises that encourage student thinking and exploration. Mathematical intuition should be nurtured, and students should naturally understand that mathematics emerges from their everyday experience. Group problem-solving projects should be encouraged as students look more to their peers and the technologies available to them in finding solutions. The hope, of course, is that success patterns will increase and many of the anxious attitudes students have towards mathematics will improve.

Rhetoric alone will not produce successful change. The proficiencies should be supported by strong staff development programs in both content and process. Massive efforts should also be made to more fully integrate technology into mathematics classrooms if our high expectations for students are to be realized.

The result of these efforts is an opportunity for the State of New Jersey to share a curricular course in mathematics education that will guide its students into the next century. The core course proficiencies will serve as a guideline for local districts as they review their course requirements in these areas. As such, they should help promote greater equity among districts and also foster higher standards of performance among students.

CORE COURSE PROFICIENCIES:

Algebra

ALGEBRA I

Introduction

Algebra is the language through which most of mathematics is communicated and therefore is a fundamental lifetime skill. Problem solving is inherent to doing mathematics. It is much more than applying specific techniques to the solution of classes of word problems; it is an integral part of mathematics. Therefore, rather than separating problem solving as a cluster unto itself, it has been integrated throughout all clusters. In order to encourage investigation, cooperation, and communication, instructional settings should foster problem posing, as well as problem solving.

Given the present state of technology, instruction throughout algebra should capitalize on both calculator and computer capabilities. Specifically, students should learn to use the calculator as a tool for processing data and performing calculations to investigate and solve problems where appropriate. This technology enables schools to provide a richer and more realistic set of algebra experiences for all students.

Algebra I makes the transition from the specifics of arithmetic to the generalizations of higher mathematics.

Examples are included as needed to clarify the intent of a proficiency and to distinguish between the two levels of algebra. They are not intended to limit the range of assessment items. Knowledge of the following formulas is assumed:

$c^2 = a^2 + b^2$

$d = rt$

$A = lw$

$P = 2l + 2w$

$i = prt$

$C = \pi d$

$A = \pi r^2$

ALGEBRA I

Cluster #1: Equations/Inequalities

The power of algebra derives from the use of variables in equations and inequalities to represent both real-world and mathematical situations. Graphical representations of equations and inequalities provide an effective means of summarizing data and furnish the basis for interpolating and extrapolating values not included in the original data. The most useful and most used skill in all of algebra is the ability to solve different types of equations and inequalities.

Students will be able to:

1.1 Understand the process of solving linear equations and inequalities in one variable and, where appropriate, represent their solutions graphically.

Example: Solve for x: $5x - 3 = 7$.

Example: Solve for x: $2(x - 3) = 7x + 4$.

Example: Solve for x: $\frac{1}{2}x + \frac{2}{3} = 3$.

Example: Which of the following equations are always true? Never true? Sometimes true?

A) $2 + x = x + 2$
B) $x + y = 15$
C) $x - 3 = x + 4$

Example: Explain why 7 is not a solution to the equation $3x + 4 = 10$.

Example: Solve for x: $ax - b = c$.

Example: Solve and graph the solution to: $2 - x < 3$ on a number line.

Example: Solve. David paid an import tax of $160 on a $25,000 car. The tax varies directly as the cost of the car. At that rate, how much import tax would he owe on a car that cost $17,000?

1.2 Understand the process of solving quadratic equations for which knowledge of completing the square or the quadratic formula is not required.

Example: Solve for x: $x(x - 1) = 0$.

Example: Solve for x: $x^2 + 2 = 3x$.

Example: Solve for x: $x^2 = 7$.

1.3 Understand the process of solving systems of linear equations in two variables by elimination, substitution, or graphing.

Example: Solve for x and y:

$$x - y = 6$$
$$3y + 5 = 2x$$

Example: Which of the following is a solution of the system?

$x + y = 7$

$2x - 3y = 4$

A) (5, 0)

B) (2, 5)

C) (5, 2)

D) (3, 4)

Example: Interpret a system from a graph. (Note: A system of linear equations and an appropriate graph to be given.)

Example: Which of the following are solutions of $2x - y = 5$?

A) (0, 5)

B) (1, 3)

C) (4, 4)

D) (0, -5)

1.4 Apply the process of solving problems using equations and inequalities.

Example: You save $24.00 on an item which is on sale for 1/3 off. What was the original selling price of the item?

Example: At the Route 15 diner, a waitress earns $40.00 per day plus 75% of the tips. The cook earns $70.00 per day plus 25% of the tips. What amount of tips would result in each person getting the same total number of dollars per day?

ALGEBRA I

Cluster #2: Number Systems

Students should understand informally that mathematical structures, such as the real number system, provide a strong foundation and an organizational system for a variety of content strands which they will encounter in their study of mathematics.

Students will be able to:

2.1 Understand and apply the field properties of the real numbers, including operations with signed numbers.

Example: I. $A(B + C) = AB + AC$

II. $A - B = B - A$

III. $AB = BA$

For all real numbers A, B, and C, which are always true?

a) Only I
b) Only I and II
c) Only I and III
d) I, II, and III

Example: Simplify $42 - 3[8 \div (5 - 3) + 7]$.

2.2 Represent, order, and use numbers in a variety of equivalent forms (integers, fractions, decimals, percent, square root, exponential/ scientific notation, calculator display).

Example: There are about 260 million people in the United States. Assume that each person eats eight pounds of sugar a year and that sugar costs $.50 per pound. How much money can the sugar industry expect to take in each year from selling sugar? Give the answer in scientific notation and in words.

Example: Explain why the product of four negative numbers is a positive number.

Example: Make a decimal estimate of the square root of 75.

ALGEBRA I

Cluster #3: Language of Algebra

As a prerequisite to virtually all of mathematics, students should understand that algebraic symbols, such as variables, may represent objects rather than merely numbers. This understanding will provide a foundation for such essential skills as using computers or calculators to provide equivalent representations, both numeric and graphical, for equations, inequalities, and their solutions.

Students will be able to:

3.1 Understand the concept and use of variables.

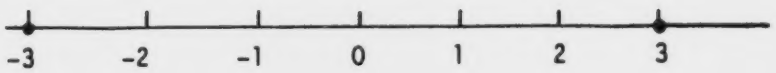
Example: $x = 5$

$y = -4$.

Evaluate $2x - 3y$.

3.2 Understand and use mathematical vocabulary and symbolic notation to represent and communicate ideas.

Example: Which equation describes this graph?



A) $x = 3$

B) $x > 3$

C) $|x| = 3$

D) $|x| \leq 3$

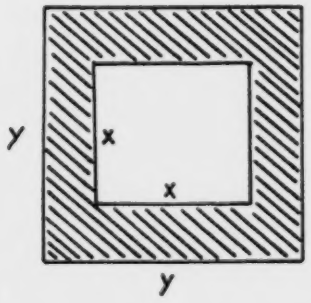
Example: Express in simpler terms $(3x^2)^4$.

Example: Given the following set of data, calculate the mean, median, and mode. (Note: A set of data is to be given.)

3.3 Formulate, simplify, and evaluate algebraic expressions

Example: Express the cost of P pounds of potatoes at 70 cents a pound.

Example: Write an expression for the shaded area between the two squares.



ALGEBRA I

Cluster #4: Algebraic Expressions

To facilitate problem solving and their further study of mathematics, students must develop facility in formulating and manipulating algebraic expressions.

Students will be able to:

4.1 Perform operations on and simplify polynomial, rational, and square root expressions

Example: Simplify

$$(3x^2 + 2x + 5) - 3(x^2 - x + 4).$$

Example: Simplify

$$\frac{3}{x + 1} - \frac{2}{x}$$

Example: Express in simpler form

$$\sqrt{20x^2} \quad \text{for } x \geq 0.$$

4.2 Apply skill in factoring polynomial expressions

Example: Factor completely $3x^2 - 27$.

Example: Factor completely $2x^2 - 11x + 15$.

Example: Write all factorable quadratic trinomials whose first term is x^2 and whose last term is -10.

Example: Show $(x + y)^2 \neq x^2 + y^2$.

4.3 Use algebraic expressions to represent mathematical and real-world situations.

Example: At the Prince Hotel, a room costs \$79 per day, a sightseeing trip cost \$15 each, and the average cost of each meal is \$12. Let

D = number of days,

T = number of trips,

M = number of meals.

Represent the cost on a vacation for these three items.

ALGEBRA I

Cluster #5: Relations, Functions, and Graphing

The function concept is important because it is a mathematical representation of many input-output situations in the real world, including those that have arisen as a result of technological advances. It is a key unifying idea in mathematics. Graphing, essential to the study of algebra, allows students to visualize symbolic representations, including relations and functions.

Students will be able to:

5.1 Understand the concept and use of independent and dependent variables.

Example: For $y = 2x$, explain the relationship between x and y.

Example: From the graph of $y = 2x$, explain the relationship between x and y.

Example: The number of calories in a container of milk **(M)** varies directly with the amount of milk in the container **(C)**. If a 10-ounce glass contains 180 calories, which of the following equations expresses the relationship between calories and ounces of milk?

A) $M = 18C$

B) $C = 180M$

C) $C = 18M$

D) $10C = M$

5.2 Represent and analyze linear relationships among a table of values, an algebraic formula, a written statement, and a graph, including the concepts of slope and intercept.

Example: For the function $y = |x|$:

A) Calculate the value of y for each integer value of x, starting at $x = -6$ and ending at $x = 8$.

B) Plot the points and describe the pattern.

5.3 Construct, interpret, and/or draw inferences from graphs, tables, and charts that reflect data from real-world situations.

ALGEBRA II

Introduction

Algebra is the language through which most of mathematics is communicated and therefore is a fundamental lifetime skill. Problem solving is inherent to doing mathematics. It is much more than applying specific techniques to the solution of classes of word problems; it is an integral part of mathematics. Therefore, rather than separating problem solving as a cluster unto itself; it has been integrated throughout all clusters. In order to encourage investigation, cooperation, and communication, instructional settings should foster problem posing, as well as problem solving.

Given the present state of technology, instruction throughout algebra should capitalize on both calculator and computer capabilities. Specifically, students should learn to use the calculator as a tool for processing data and performing calculations to investigate and solve problems where appropriate. This technology enables schools to provide a richer and more realistic set of algebra experiences for all students.

Algebra II extends the fundamental concepts and skills of elementary algebra to a higher level and, while introducing new concepts, draws upon the same basic notions previously studied.

Examples, included as needed to clarify the intent of a proficiency and to distinguish between the two levels of algebra, are not intended to limit the range of assessment items. Knowledge of the following formulas is assumed:

Quadratic Formula

Midpoint Formula

Distance Formula

Slope

Slope-Intercept Equation of a Line.

ALGEBRA II

Cluster #1: Equations and Inequalities

The power of algebra derives from the use of variables in equations and inequalities to represent both real-world and mathematical situations. Graphical representations of equations and inequalities provide an effective means of summarizing data and furnish the basis for interpolating and extrapolating values not included in the original data.

Students will be able to:

1.1 Understand the process of solving equations and inequalities with one variable and, where appropriate, representing solutions graphically.

Example: Solve $\frac{1}{x} + \frac{2}{3} = \frac{7}{x}$.

Example: Which of the following equations are always true? (Note: A set of equations to be given.)

Example: Explain why -2 is not a solution of $-5 < 2 - x < 3$.

Example: Solve $\frac{7}{x - 5} - 2 = \frac{x + 2}{x - 5}$.

Example: Solve $\sqrt{x - 2} = 3$.

1.2 Understand the process of solving quadratic equations.

Example: Solve $y^2 - 4y = 3$.

Example: Solve $\frac{2}{x - 1} + x = 4$.

Example: Explain the process of solving

$$x^3 + x^2 + x = 0.$$

Example: $F(x) = 3x^2 + 3x - 1$.

Approximate x so that $F(x) = 0$; $F(x) = 2$.

1.3 Understand the process of solving systems of equations in two variables.

Example: Solve
$$x + y = 7$$
$$x^2 + y^2 = 25.$$

Example: Given that F and C are linearly related and that F = 32 when C = 0 and F = 212 when C = 100, derive the relationship between F and C.

1.4 Understand the process of solving systems of linear inequalities in two variables by graphing.

Example: Given the system

$$y > x$$
$$y < 2 - x,$$

Which of the following are solutions of the system? (0,0), (-2,5), (2,-4), (-2,0).

1.5 Understand the process of solving absolute value equations and inequalities.

Example: Solve $|x - 4| \geq 6$.

1.6 Apply the process of solving problems using equations and inequalities.

Example: Tickets for a dinner-dance were \$20 for a single ticket or \$35 for a couple. If 128 people attended the event, and ticket sales totaled \$2,280, how many single tickets and how many couples tickets were sold? Select the appropriate system of equations for solving the above problem. (Note: A set of systems of equations to be given.)

Example: Given $H = RT - 4.9T^2$, find the values of T corresponding to $H = 0$ and $R = 5.88$.

ALGEBRA II

Cluster #2: Number Systems

Students should understand informally that mathematics structures, such as real, rational, and complex numbers provide a strong foundation and an organizational system for a variety of content strands which they will encounter in their study of mathematics.

Students will be able to:

2.1 Understand and apply the field properties of the complex numbers, including operations with complex numbers.

Example: Solve for x and y.

$$3 + xi = y + 2i$$

Example: Determine the product $(2 + 3i)(4 - 3i)$.

Example: Determine the product $(2 + \sqrt{3})^2$.

2.2 Represent, order, and use numbers in a variety of equivalent forms (fractions, decimals, percent, exponential/scientific notation, radical, rational exponents, calculator display).

Example: Express 4^3 as a power of 2.

Example: Estimate the value of y.

$$y = (9.4 \times 10^5) \div (3.6 \times 10^{-3})$$

ALGEBRA II

Cluster #3: Language of Algebra

As a prerequisite to virtually all of mathematics, students should understand that algebraic symbols, such as variables, may represent objects rather than merely numbers. This understanding will provide a foundation for such essential skills as using computers or calculators to provide equivalent representations, both numeric and graphical, for equations, inequalities, and their solutions.

3.1 Understand the concept and use of variables.

Example: $x = 2p + 1$

$y = 4p - 3.$

Express $x + y$ in terms of p.

3.2 Understand and use mathematical vocabulary and symbolic notation to represent and communicate ideas.

Example: $F(x) = 3x^2 - 2x + 5$. Find $F(x + 2)$.

Example: A cost/profit relationship.

3.3 Formulate, simplify, and evaluate algebraic expressions.

Example: A square pool with side x feet is surrounded by a concrete walkway with a uniform width of two feet. Express the area of the walkway in terms of x.

ALGEBRA II

Cluster #4: Algebraic Expressions

To facilitate problem solving and their further study of mathematics, students must develop facility in formulating and manipulating algebraic expressions.

Students will be able to:

4.1 Perform operations and simplify polynomial, rational, and radical expressions and expressions with rational exponents.

Example: Simplify $\dfrac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x^2}}$

Example: Simplify $(x^2 - y^2) - (x - y)^2$.

Example: Simplify $\dfrac{5x^2}{x - 1} - \dfrac{3x + 2}{x^2 - 1}$

Example: Simplify $\dfrac{2}{\sqrt{3} - 1}$

Example: Which of the following is a reasonable approximation of $\sqrt{2}$?
(Note: A number of values is to be given.)

4.2 Factor polynomial expressions.

Example: Factor completely $x^3 - 64$.

Example: Factor completely $(x + y)^2 - 16$.

Example: Factor completely $x^3 - x^2 + x - 1$.

Example: Which of the following rational expressions are reducible? (Note: A set of rational expressions is to be given.)

4.3 Use algebraic expressions to represent real-world situations.

Example: A forester plants 12,000 trees in the spring. Suppose that each tree contains 1/50 of a cord of wood when it is planted and that each year the amount of wood grows by a scale factor of x. How many cords of wood are in the forest at the end of the third spring?

ALGEBRA II

Cluster #5: Relations, Functions, and Graphing

The function concept is important because it is a mathematical representation of many input-output situations found in the real world, including those that have arisen as a result of technological advances. It is a key unifying idea in mathematics. Graphing is essential to the study of algebra. Graphs allow students to visualize symbolic representations, including relations and functions.

Students will be able to:

5.1 Understand and apply the concept of finite and infinite relations and functions (including domain and range).

Example: Given the graph of a relation (finite or infinite), determine if it is a function.

5.2 Represent and analyze the relationship among a table of values, an algebraic formula, a written statement, and a graph, including the line, the parabola, and the circle.

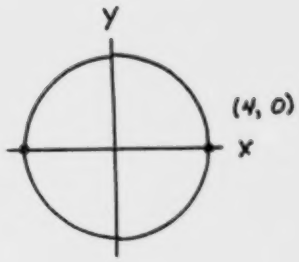
Example: $x = 2$ is a solution to the equation

$$x^2 + 2x - 8 = 0.$$

How is this related to the graph of the function

$y = x^2 + 2x - 8$?

Example: Which of the following describes the graph?



A) $x^2 + y^2 = 16$

B) $x^2 + y = 4$

C) $x + y = 4$

D) $x^2 - y^2 = 16$

5.3 Understand operations on, and the general properties and behavior of, functions, including the basic concepts of exponential and logarithmic functions.

Example: Find g [f(x)].

$f(x) = x + 1$

$g(x) = x^2$

Example: The volume of a fixed amount of gas (at constant temperature) is inversely proportional to the pressure of the gas. Express volume in terms of pressure if a pressure of 36 pounds per square inch compresses the gas to a volume of 300 cubic feet. Could you compress the gas to zero volume? Explain.

Example: Which of the following is equivalent to $\log_2 (32)$? (Note: Several values are to be given.)

5.4 Apply the concepts and skills of coordinate geometry, such as midpoint, distance, and slope.

Example: Find the value of K so that the line joining points $P_1(4,2)$ and $P_2(6,K)$ has a slope of 5/2.

Example: Determine if the point P(2,3) is on, inside, or outside the circle $(x - 2)^2 + y^2 = 36$.

5.5 Construct, interpret, and/or draw inferences from graphs, tables, and charts that reflect data from real-world situations (to include evaluating arguments of other persons).

Example: Jacob makes pine end tables. He uses $y = 80x + 100$ as the cost formula for producing x tables. Each table sells for \$100, so he uses $y = 100x$ to determine income. How many tables must he sell to break even? (Note: An appropriate graph is to be included.)

CORE COURSE PROFICIENCIES:

Geometry

GEOMETRY

Preface

Geometry has historically held an important role in high school mathematics, primarily through its focus on deductive reasoning and proof. In addition, geometry helps us represent and describe the world in which we live; it includes categorizations and properties of shapes and their relationships. Developing skills in deductive reasoning, learning how to construct proofs, and understanding geometric properties are important outcomes of the high school geometry course. Equally important, however, is the continued development of visualization skills, pictorial representations, and applications of geometric ideas to describe and answer questions about natural, physical, and social phenomena. The core curriculum for geometry described here addresses all of these goals for all students.

Deductive reasoning is highly dependent upon communication skills. In fact, mathematics can be considered as a language - a language of patterns. This language of mathematics must be meaningful if students are to discuss mathematics, write proofs in a variety of ways, and apply geometry productively. Communication and language play a critical role in helping students to construct links between their informal, intuitive geometric notions and the more abstract language and symbolism of high school geometry.

Geometry describes the real world from several viewpoints. One viewpoint is that of standard Euclidean geometry - a deductive system developed from basic axioms. Other viewpoints, widely used internationally, are those of

coordinate geometry and transformational geometry. In order for these alternative viewpoints to be explored, students enrolled in geometry classes must have successfully completed a first-year algebra course. The interplay between geometry and algebra strengthens students' ability to formulate and analyze problems from situations both within and outside mathematics. Although students will at times work separately in synthetic, coordinate, and transformational geometry, they should also have many opportunities to compare, contrast, and translate among these systems. Further, students should learn that specific problems are often solved more easily in one or another of these systems.

Pictorial representation and visualization are also important aspects of a high school geometry course. Students should have opportunities to visualize and work with two- and three-dimensional figures in order to develop spatial skills fundamental to everyday life and to many careers. By using physical models and other real-world objects, students can develop a strong base for geometric intuition. As students work with abstract ideas, they can draw upon these experiences and intuitions.

The goals of high school geometry include applying geometric ideas to real problems in a variety of areas. Each student must develop the ability to solve problems if he or she is to become a productive citizen. Hence, instruction in geometry classes must begin with problem situations -- not only relatively simple exercises to be accomplished independently but also problems to be solved in small groups or by the entire class working cooperatively.

Applications of mathematics have changed dramatically over the last twenty years, primarily due to rapid advances in technology. Geometry has, in fact, become more pertinent to students because of computer graphics. Thus, in teaching geometry, we must consider the appropriate role of computers and calculators. In order to teach geometry effectively in New Jersey's high schools, it is important that:

- appropriate calculators be available to all students at all times;
- a computer be available in every classroom for demonstration purposes;
- every student have access to a computer for individual and group work;
- students learn to use the computer as a tool for processing geometric information, for generating geometric figures, and for performing calculations to investigate and solve problems.

We recognize, however, that access to technology is no guarantee that any student will become mathematically literate. We believe that calculators should be used in instruction and testing in the high school geometry course. We further believe that computers should be used for teacher demonstration and student exploration. Due to the cost and present lack of availability of computers, however, their use in testing is not recommended at this time.

In developing the following core proficiencies for geometry, we have thus included several general themes: problem solving and higher-order thinking, communication, connecting geometry and algebra, spatial visualization, using concrete models and manipulatives, estimation and practical applications, and using calculators and computers. Although these themes are not core proficiencies in and of themselves, they are reflected as important emphases in the examples given to illustrate the scope of each of the core proficiencies.

The examples included are two types: instructional and assessment. Instructional examples suggest classroom activities appropriate for addressing the proficiency. Some of these involve group and/or computer exploration. Assessment examples are those that may be used either in instruction or testing.

Core Proficiencies for Geometry*

I. Relationships of points, lines, planes, and space

Introduction: High school geometry should build on the strong conceptual foundation students developed in the K-9 programs. Students should have opportunities to visualize and work with two- and three-dimensional figures in order to develop skills fundamental to everyday life and to many careers. Physical models and other real-world objects should be used to provide a strong base for the development of students' geometric intuition so that they can draw on these experiences in their work with abstract ideas.

Students will be able to:

1. Interpret and draw two- and three-dimensional objects.

   Examples:

   a. Describe and sketch the possible intersections of two planes. (Instructional)

   b. Use six toothpicks to create four equilateral triangles. (Instructional)

* Please note that instructional examples suggest classroom activities while assessment examples may be used in either instruction or testing.

c. Describe the figure that is formed by rotating a right triangle about one leg. (Assessment)

d. Sketch a quadrilateral whose diagonals are perpendicular bisectors of each other. (Assessment)

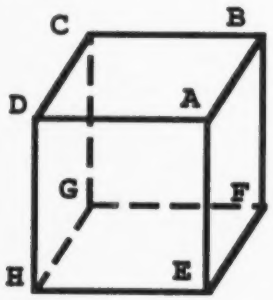
2. Understand and apply the basic concepts related to points, lines, planes, and space, including parallelism and perpendicularity.

Examples:

a. Make models to investigate the possible outcomes of slicing a cube with a plane. (Group exploration) (Instructional)

b. Which is more stable: A three- or four-legged stool? Justify your answer. (Assessment)

c. Given a cube ABCDEFGH, name at least one pair of parallel faces, one pair of parallel edges, one pair of perpendicular faces, and one pair of skew edges. (Instructional) (Assessment with picture)



II. Systems of measurement and units of measurement

Introduction: The study of measurement shows the usefulness and practical application of mathematics. Measurement enables students to describe the real world in a quantitative way. Students should be able to identify attributes, select appropriate units, apply measurement concepts, and communicate ideas related to measurement.

Students will be able to:

1. Demonstrate an understanding of figures based upon information regarding angle and/or side measure.

   Examples:

   a. If the measure of two sides of a triangle are 5 and 11, what is the range of possible measures for the third side? (Instructional or Assessment)

   b. I am an equiangular quadrilateral. What special kind of quadrilateral am I? Justify your response. (Assessment)

   c. Draw a scalene triangle. (Assessment)

   d. Construct a right isosceles triangle. (Assessment)

e. Construct a regular hexagon. (Instructional)

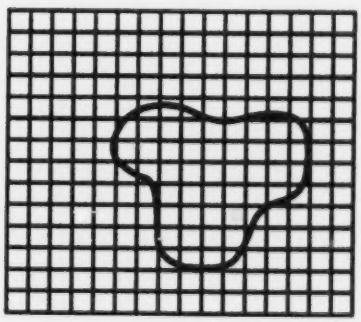
f. Explain why a triangle cannot have two obtuse angles. (Assessment)

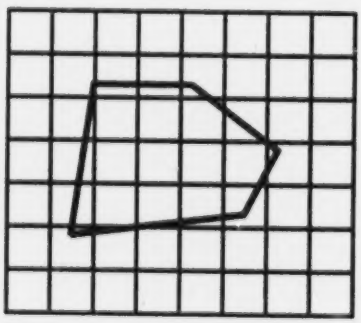
g. Draw a triangle with sides of 5, 12, and 13 centimeters. Describe the figure as fully as you can. (Instructional)

2. Understand and apply the concepts of perimeter, circumference, area, surface area, and volume.

Examples:

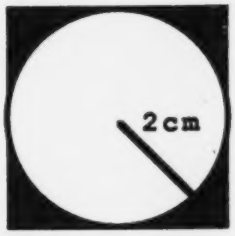
a. Estimate the area of the enclosed regions. (Instructional or Assessment)





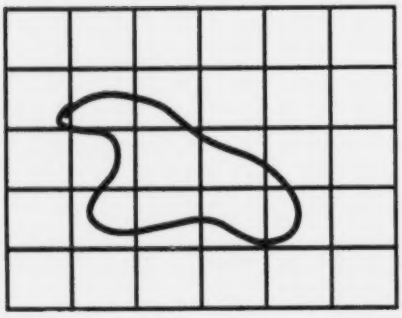
b. Given a rectangular solid with dimensions 3, 7, and 8 inches, what is the ratio of the new volume to the original volume if three inches is added to each dimension? (Instructional)

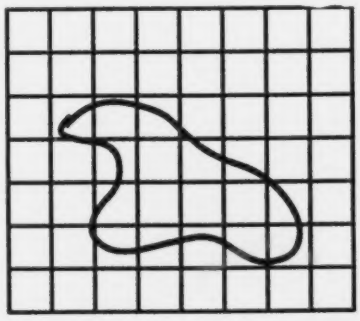
c. A circle is inscribed in a square.
Find the area of the shaded region.
(Instructional or Assessment)

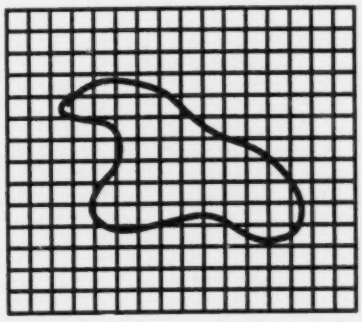


d. Given a 10" x 15" rectangular piece of metal, determine the size of the squares that must be removed from the corners in order to make an open box of maximum volume. (Computer, calculator, and/or group explorations) (Instructional)

e. Choose the grid from which to make the best estimate of the area of the enclosed region. (Assessment)

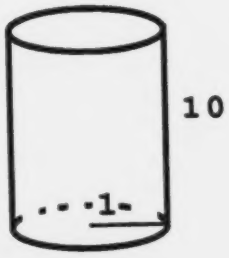






f. Given a right circular cylinder with radius 1 inch and height 10 inches, find the total surface area.

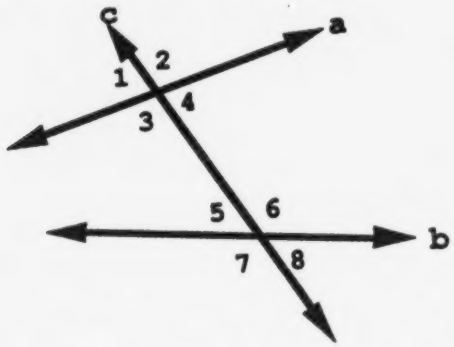
($SA = 2\pi rh + 2\pi r^2$; $\pi = 3.14$) (Assessment)



3. <u>Understand and apply concepts of angle measure, including angle pair relationships</u>.

Examples:

a. Given that line a and line b are not parallel and that line c is a transversal,

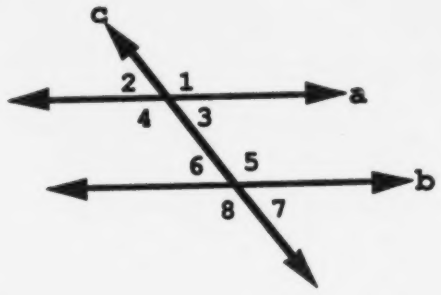


give an example of each of the following:

vertical angles
alternate interior angles
supplementary angles
interior angles on the same side of the transversal
(Assessment)

b. Two angles are complementary. The measure of one angle is 20 less than twice the measure of the other angle. Find the measure of each angle. (Assessment)

c. In the diagram below, a $\parallel$ b, $m\angle 3 = 3x - 4$, and $m\angle 6 = 2x + 1$. Find the measurement of all numbered angles. (Assessment)



III. Reasoning - deductive, inductive

Introduction: Although the deductive nature of geometry first developed by the Greeks should not be overlooked, the interplay between inductive and deductive experiences should be emphasized. Making conjectures, gathering evidence, and building an argument to support such notions are fundamental to mathematics.

Students will be able to:

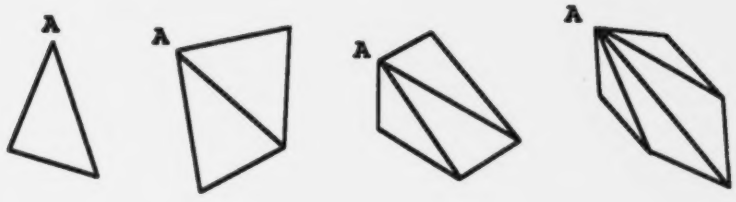
1. Make and test a conjecture.

Examples:

a. Investigate the results of connecting, in order, the midpoints of the sides of a quadrilateral. State one conjecture based

on your investigation. (Computer and/or group exploration) (Instructional)

b. How is the number of diagonals drawn from one vertex of a polygon related to the number of sides of that polygon? (Assessment)



2. Judge validity of an argument.

Examples:

a. If line a is perpendicular to line b and line b is perpendicular to line c, then line a is perpendicular to line c. Is this a valid or invalid argument? Support your answer. (Instructional or Assessment)

b. All squares are rectangles.
ABCD is a rectangle.
Therefore, ABCD is a square.

Is this a valid argument? Support your answer. (Instructional or Assessment)

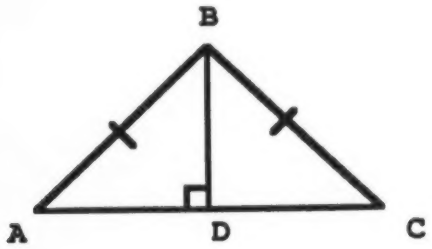
c. If the sun shines, we will go to the zoo. The sun is not shining, therefore, we are not going to the zoo. Is this a valid argument? Support your answer. (Instructional or Assessment)

3. Construct a simple valid argument.

Examples:

a. Provide an argument that the sum of measures of the interior angles of a triangle is 180°. (Hint: Draw a line through a vertex parallel to the opposite side.) (Instructional)

b. Write a paragraph proving that triangle ABD is congruent to triangle CBD. (Assessment)



4. Formulate a counter example.

Examples:

a. Provide an example to demonstrate that SSA does not prove two triangles congruent. (Instructional)

b. Show that all isosceles triangles are not equilateral. (Assessment)

c. If two line segments intersect, then they bisect each other. True or False. Defend your answer. (Assessment)

IV. Coordinate Geometry

Introduction: The connections between algebra and geometry are among the most important in high school mathematics. Coordinate geometry provides tools for students to use in solving a wide range of problems.

Students will be able to:

1. Apply the principles of coordinate geometry to determine properties of geometric figures.

Examples:

a. Given three points N(3,4), E(9,4), and A(12,8) in a plane, determine the coordinates of the 4th point T to create a parallelogram NEAT. (Assessment)

b. Given the points F(0,0), U(6,0), and N(6,8), find the midpoint of each side of triangle FUN and determine what figure is formed by joining the midpoints. Justify your answer. Be specific. (Assessment)

c. Given points M(4,4), A(-1, -6), T(-2, -2), H(1, 4). If the points are connected in order, what shape is figure MATH? Be specific. Justify your answer. (Assessment)

d. Find the area of the figure bounded by the x and y axes and the line $y = -2x + 3$. (Group exploration) (Instructional)

V. Congruency

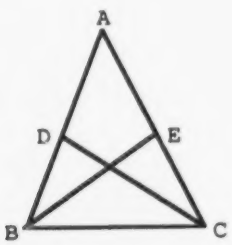
Introduction: Congruence, historically a major topic in high school geometry, provides a convenient focus for developing skills in writing proofs. Since deductive reasoning is addressed in other areas, such as transformations and coordinate geometry, less time and emphasis on proof of congruence is required.

Students will be able to:

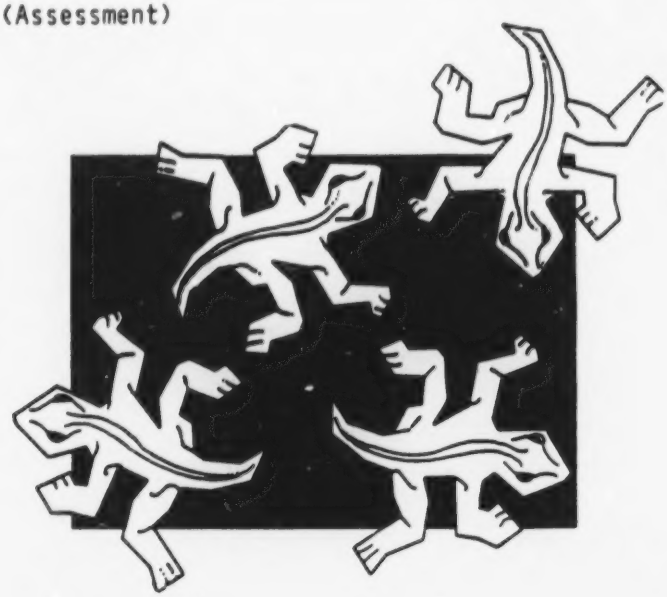
1. Demonstrate the ability to identify congruent figures and understand and use their properties in practical applications.

   Examples:

   a. Given $\overline{AB} \cong \overline{AC}$ and $\overline{AD} \cong \overline{AE}$, name a pair of $\cong$ $\triangle$s and justify your answer. (Assessment).



   b. The four reptiles shown here are congruent, yet one is different from the others in a very basic way. Which reptile is it and why? (Assessment)



c. You need to plant a tree perpendicular to level ground. You have 120 feet of wire, wire cutters, and stakes available. How can you use these materials to make sure the tree is perpendicular? Justify your answer. (Instructional)

VI. Properties of polygons and circles

Introduction: The study of properties of polygons and circles provides opportunities for students to apply mathematics to real-world situations in such areas as construction, art, and the sciences. Formalizing notions of shape and analyzing attributes of polygons and circles are important goals of a geometry course. This provides experiences that deepen students' understanding of shapes and their properties, with an emphasis on their wide applicability in human activity.

Students will be able to:

1. Understand and apply properties of lines and segments related to triangles.

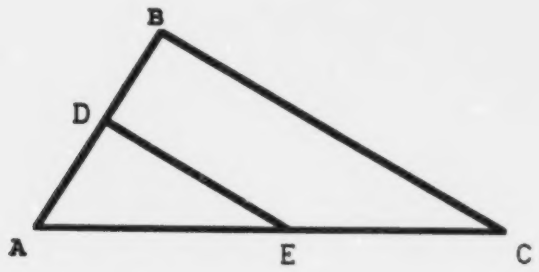
Examples:

a. The Widget Company of Flatland wishes to service three communities. Where should the company build their office in order to be equidistant from all three communities? (Instructional)

b. Fold a triangle to show that the angle bisectors meet at one point. (Instructional)

c. Investigate the properties and make conjectures concerning the medians of a triangle. (Computer and/or group activity) (Instructional)

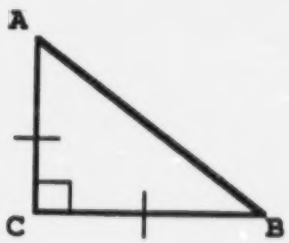
d. Given triangle ABC with D and E midpoints of $\overline{AB}$ and $\overline{AC}$, respectively, find BC if DE = 7.5 centimeters. (Assessment)



2. Apply the Pythagorean Theorem.

Examples:

a. If AC = 5, find AB. (Assessment)



b. If the lengths of the diagonals of a rhombus are 12 and 16, what is the perimeter of the rhombus? (Assessment)

c. Can a round table top with a diameter of 9 ft. be moved through a door opening 8 ft. by 4 ft.? You need not consider the thickness of the table top. Justify your answer. (Instructional)

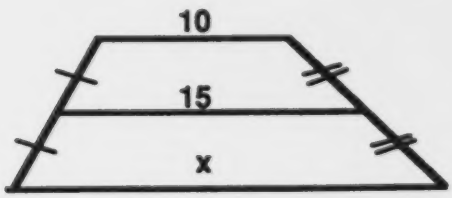
d. Find the distance between P(1, 1) and Q(6,13). (Assessment)

e. Given an equilateral triangle with a side length of six, find the length of the altitude. (Instructional or Assessment)

3. Understand and apply concepts and principles relating to quadrilaterals and other polygons.

Examples:

a. Given the trapezoid shown, find x. (Assessment)

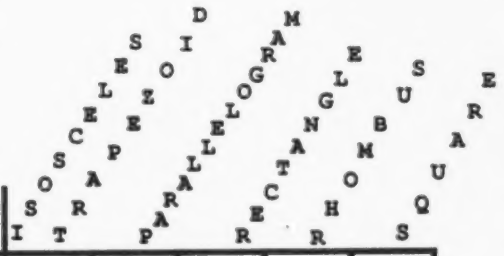


b. Find the sum of the measures of the interior angles of a 14-sided polygon. (Assessment)

c. The measure of an exterior angle of a regular polygon is 45°. What is the degree measure of one interior angle, and how many sides does the polygon have? (Assessment)

d. Sam wishes to make a regular hexagonal flower bed. He has six railroad ties each 8 ft. long. Describe how to position the railroad ties to form the desired flower bed. (Instructional)

e. Check the box to indicate if the property applies to the given quadrilateral. (Assessment)

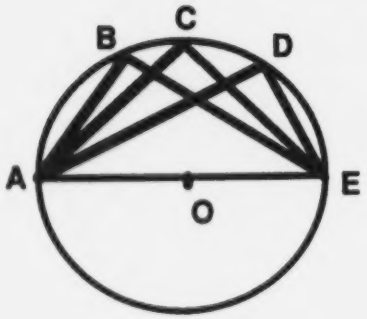


PROPERTY					
Congruent Diagonals					
Perpendicular diagonals					
Opposite sides parallel					
Consecutive angles supplementary					
Consecutive sides congruent					

4. Understand and apply proprieties of lines, segments, angles, and arcs related to circles.

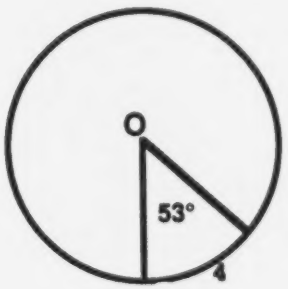
Examples:

a. What do angles B, C, and D of circle O have in common? (Assessment)



b. A machinist needs to replace a circular disc. All he has to work with is a piece of that disc. How can he find the original diameter using this piece? (Group exploration) (Instructional)

c. Given the central angle measure and its arc length, find the measure of the circumference of the circle O below. (Calculator) (Instructional or Assessment)



d. If the sides of a triangle inscribed in a circle cut off arcs in the ratio of 2:3:5, find the measure of the smallest angle of the triangle. (Instructional or Assessment)

e. Draw a diagram of two concentric circles, where a chord of the larger circle is tangent to the small circle. (Assessment)

VII. Transformations

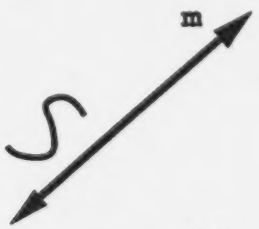
Introduction: The study of geometry through the use of transformations--the geometric counterpart of functions--has changed geometry from a static to a dynamic subject. Transformations provide great additional power that can be used, for example, to describe and produce moving figures on a video screen.

Students will be able to:

1. Understand and apply simple reflections, rotations, and translations, recognizing their effects on geometric figures.

Examples:

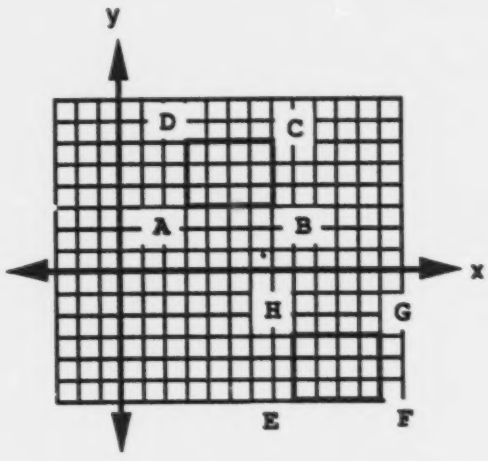
a. Sketch the image of S through a reflection over line m.



b. Identify all the lines of symmetry in the figure at the right. Through how many degrees should the figure be rotated so that it and its rotation image coincide. (Instructional or Assessment)

c. What is the image of the point P(-2, 3) through a reflection over the y axis? (Assessment)

d. If EFGH is the image of ABCD under a translation, what would be the image of the origin under this transformation? (Assessment)



e. Discuss the ways in which a figure and its image under a rotation are alike and different. (Instructional)

VIII. Similarity

Introduction: Measuring and comparing the sides and angles of similar polygons help students develop and understand the mathematical concepts of similar figure. Similarity also can be related to such real-world contexts as photographs, models, projections of pictures, and photocopy machines.

Students will be able to:

1. Identify similar figures and understand and apply their properties.

   Examples:

   a. Mary wants to measure the height of a tree. She places a mirror 10 feet from the base of the tree and walks backward until she sees the top of the tree in the mirror. She is now 15 feet from the base of the tree. If Mary is 4' 6" tall, what is the height of the tree? (Instructional or Assessment)

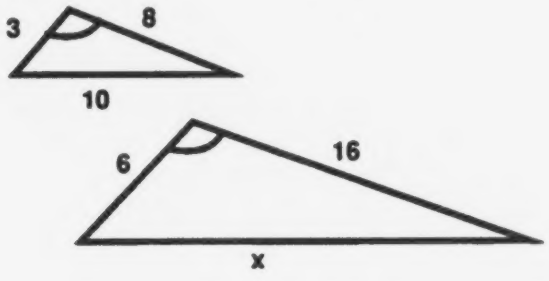
b. Use a computer graphics program to generate a figure similar to a given figure. Investigate the ways in which the two figures are alike and different. How are the volume, perimeter, and area affected? (Computer exploration) (Instructional)

c. Draw a figure that has all sides twice as large as the given parallelogram. What happens to the area? Justify your answer. (Instructional)

d. Explain why all cylinders are not similar. (Assessment)

e. Explain why all equilateral triangles are similar. (Assessment)

f. Given these two triangles:



Explain why the two triangles are similar. Find x. What is the ratio of the areas? What is the ratio of the perimeters? (Assessment)

IX. Symbolism and Notation

Introduction: Students' ability to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking.

A table similar to the one below will be recommended for inclusion on the assessment so that students will understand the symbolism.

SYMBOL	MEANING	SYMBOL	MEANING
$\overleftrightarrow{AB}$	line AB	$\cup$	union
$\overline{AB}$	segment AB	$\cap$	intersection
AB	distance from A to B	$\subset$	subset
$\overrightarrow{AB}$	ray AB	$\emptyset$	empty set
$\angle A$	angle A	$\parallel$	is parallel to
$m\angle A$	measure of $\angle A$	$\perp$	is perpendicular to
$\sim$	is similar to	$\cong$	is congruent to

Students will be able to:

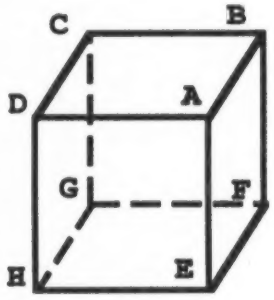
1. Use and interpret appropriate symbols and notation to communicate geometric ideas.

Examples:

a. Draw a diagram to show $\overline{AB} \cap \overline{CD} = \emptyset$. (Assessment)

b. $\triangle ABC \sim \triangle DEF$. List corresponding angles and corresponding sides. (Assessment)

c. Given plane M intersecting cube ABCDEFGH, draw, label, and identify possible intersections. (Group explorations) (Instructional)



CORE COURSE PROFICIENCIES:

General Mathematics

GENERAL MATHEMATICS

Preface

The general mathematics core proficiencies were designed to provide a basic understanding of fundamental mathematical ideas and their applications to support development of students' skills for the Eleventh-grade High School Proficiency Test. It is assumed that students will enter these courses with existing knowledge and skills covered by the current ninth-grade HSPT and the future eighth-grade test. Certain proficiencies were included in general mathematics to enable students to exit from this program and successfully enter the algebra/geometry sequence. Proficiencies in earlier courses will be treated as assumed knowledge in later courses. Examples are included to clarify the intent of the proficiency but are not meant to limit the range of assessment items.

Developing mathematical flexibility through the use of problem solving is the prime focus of the recommendations within the six clusters of Numerical Operations, Patterns and Functions, Data Analysis, Measurement and Geometry, Fundamentals of Algebra, and Problem Solving. The general math proficiencies provide opportunities for practical applications, spatial visualization, communication, and problem solving. Calculators must be an integral part of instruction and should be used during assessment. A card should be supplied during the assessment procedure which will include formulas, inch and centimeter ruler, protractor, and possibly a transparent grid. Emphasis should be on application, not memorization.

Also included within the sequence of proficiencies are opportunities for small group projects, discovery learning, cooperative learning, computer-based activities, use of manipulatives, and other types of modern technology.

Some students learn by visualization, some learn through manipulation of physical objects, and some learn through concrete applications in everyday life. Students learn in many different ways and need diverse experiences to maximize their learning opportunities.

CORE PROFICIENCIES FOR GENERAL MATH I

I. Numerical Operations

Students will be able to use their conceptual and procedural knowledge of numbers, numeration, and operations in real-world situations.

Students will be able to:

1. Estimate, approximate, and judge the reasonableness of answers in a variety of problem-solving situations.

   Examples: Estimate take-home pay, discounts, etc.

2. Read, write, and round numbers; and recognize place value concepts.

3. Demonstrate an understanding of primes, factors, and multiples in the solution of problems.

   Examples: Find the least common multiple, the greatest common factor and prime factorization.

4. Apply ratio, proportion, and percents in a variety of situations.

   Examples: Compare ratios, comparison shopping, simple interest, commission, etc.

5. Apply order-of-operation rules for calculator and pencil calculations.

II. Patterns and Functions

Students should be able to recognize, describe, and generalize patterns and to build mathematical models that can be used to predict the behavior of real-world phenomena that illustrate the observed pattern.

Students will be able to:

1. Use patterns and relationships to represent and solve problems using a calculator.

   Example: arithmetic progression

2. Recognize and appreciate the mathematical patterns found in nature, work place, and everyday life.

   Examples: alphabetical, numerical, geometric, and spatial relationships.

III. Data Analysis

Students should be able to collect, represent, and interpret data in ways that enable them to make meaningful decisions and to understand the

reasons for social and civic actions. As a primary outcome of this cluster, students will be able to construct and test arguments based on the validity of data and to understand the role probability plays in interpreting such arguments.

Students should have full access to calculators and computer software to assist in their study of these critical proficiencies and to ensure an efficient and thorough analysis of the data sets in this cluster.

Students will be able to:

1. Collect, organize, and describe sets of data.

   Example: Students conduct simple surveys, use reference works, and poll opinions to construct tables by characteristics and frequencies and to make appropriate bar, circle, and line graphs.

2. Explain orally and in writing results shown in tabular or graphic form.

   Example: Given a table or graphical display from a newspaper, students will summarize the results or trends shown.

3. Find the probability of a simple event.

Example: Find the probability of drawing a ball numbered with a factor of 6 from a set of balls numbered zero through 9.

4. Understand, compute, and apply the mean, median, mode, range, and rank.

IV. Measurement and Geometry

Students should understand shapes and their properties and move from concrete to abstract understanding by using physical models, two- and three-dimensional figures, real-world objects, and computer simulations.

Students will be able to:

1. Use the appropriate tool and units to estimate and measure distance, area, volume, mass, angles, and temperature.

   Examples: metric units, standard units, and conversion within systems

2. Visualize and represent two- and three-dimensional geometric figures.

3. Estimate and find the perimeter and area of irregular polygons.

4. Calculate the volume of a solid figure.

   Examples: cylinders, rectangular solids

5. Understand and apply geometric properties of plane figures, such as lines, quadrilaterals, triangles, angles, and polygons.

   Examples: congruence, similarity, parallel, perpendicular

6. Use coordinate geometry to represent relationships.

   Example: points on a map

V. Fundamentals of Algebra

Students will demonstrate both manipulative facility and conceptual understanding of algebraic principles used in problem-solving situations. Students will be able to use mathematical models, real data, graphs, tables, expressions, equations, and technological tools.

Students will be able to:

1. Set up a first-degree algebraic expression, given its equivalent literal statement and vice versa.

   Example: perimeter

2. Evaluate basic algebraic expressions.

   Examples: Evaluate $9/5C + 32$

   $a^2 + b^2$

   s^3

3. Solve linear equations.

   Examples: $2x + 6 = 10$ $\qquad$ $10\% \; x = 30$

   $\frac{1}{4}x - 7 = 12$ $\qquad$ $2x = -6$

VI. Problem Solving

Students should be able to use general strategies to analyze and solve problems. Although these strategies will be included throughout the three-year sequence, teaching a formalized process is essential at this and earlier levels.

Students will be able to:

1. Select and use a variety of techniques to analyze and solve real-world and mathematical problems.

   Examples: Manipulate physical models, draw pictures, guess and check, make charts, etc.

2. Select and apply appropriate tools needed to solve problems.

   Examples: pencil and paper, calculators, mental arithmetic, computers

CORE PROFICIENCIES FOR GENERAL MATH II

I. Numerical Operations

Students will be able to use their conceptual and procedural knowledge of numbers, numeration, and operations in real-world situations.

Students will be able to:

1. Estimate, approximate, and judge the reasonableness of answers in a variety of problem-solving situations.

   Example: Estimate the cost of a vacation, given multiple component costs.

2. Apply ratio, proportion, and percents in a variety of situations.

   Examples: compound interest, map scales, proportion

3. Understand real numbers.

   Examples: express, order, and compare

II. Patterns and Functions

Students should be able to recognize, describe, and generalize patterns and to build mathematical models that can be used to predict the behavior of real-world phenomena that illustrate the observed pattern.

Students will be able to:

1. Use patterns and functions to represent and solve problems using a calculator.

   Examples: square roots, reciprocals, compound interest

2. Determine a rule describing a given set of data.

   Example: linear relationships

III. Data Analysis

Students should be able to collect, represent, and interpret data in ways t .t enable them to make meaningful decisions and to understand social and civic actions. As a primary outcome of this cluster, students will be able to construct and test arguments based on the validity of data and understand the role probability plays in interpreting such arguments.

Students should have full access to calculators and computer software to assist in their study of these critical proficiencies and to ensure an efficient and thorough analysis of the data sets.

Students will be able to:

1. Construct and draw inferences from charts, tables, and graphs.

2. Compare two interpretations for a given display of data with respect to completeness and validity of conclusions.

   Example: Given a tabular display of the Home Run Production of four major league batters, compare two different interpretations of who is the "best" Home Run Slugger.

3. Determine the probability of a simple event by relative frequency or experimental methods.

   Example: Determine the probability of a given outcome on spinning the device 200 times.

IV. Measurement and Geometry

Students should understand shapes and their properties and move from concrete to abstract understanding by using physical models, two- and three-dimensional figures, real-world objects, and computer simulations.

Students will be able to:

1. Use coordinate geometry to represent relationships.

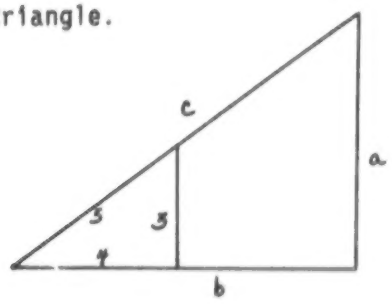
   Example: linear relationships

2. Understand transformations and their effects on geometric figures, and apply these ideas informally.

   Examples: flips, slides, turns, shrinking, and stretching

3. Classify geometric f' 'res in terms of congruence and similarity, and apply these relationships informally.

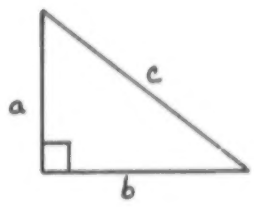
   Example: Given two similar triangles, the lengths of the sides of one, and the ratio of the corresponding sides of the two triangles, determine the lengths of the sides of the second triangle.



4. Apply the Pythagorean Theorem.

   Example: Given a = 2 and b = 3, find c.

   Given a = 3 and c = 5, find b.



V. Fundamentals of Algebra

Students will demonstrate both manipulative facility and conceptual understanding of algebraic principles used in problem-solving situations. Students will be able to use mathematical models, real data, graphs, tables, expressions, equations, and technological applications.

Students will be able to:

1. Solve equations.

   Examples: $2x + 6 = 10x$

   $\frac{1}{4}x - 7 = -3x + 3$

   $x^2 = 9$

   $x = 4$

2. Apply algebraic methods to solve a variety of problems.

   Example: Pythagorean theorem, area, volume application

CORE PROFICNENCIES FOR GENERAL MATH III

I. Numerical Operations

Students will be able to use their conceptual and procedural knowledge of numbers, numerations, and operations in real-world situations.

Students will be able to:

1. Estimate, approximate, and judge the reasonableness of answers in a variety of problem-solving situations.

   Example: Estimate production capabilities, given a margin of error (quality control).

2. Apply ratio, proportion, and percents in a variety of situations.

   Example: tax on discounted items, comparison of loan payments, other multi-step word problems.

3. Understand and apply real numbers in solving problems.

   Examples: wages, purchasing, credit, banking

II. Patterns and Functions

Students should be able to recognize, describe, and generalize patterns and to build mathematical models that can be used to predict the behavior of real-world phenomena that illustrate the observed patterns.

Students will be able to:

1. Use patterns and functions to represent and solve problems using appropriate technology.

   Example: Analyze tables using functions and relationships.

2. Describe a given set of data using a rule, verbal expression, or graph.

   Example: economic trends, census data

3. Demonstrate an understanding of arithmetic and geometric sequences and series using an intuitive/non-algebraic approach.

III. Data Analysis

Students should be able to collect, represent, and interpret data in ways that enable them to make meaningful decisions and understand the reasons for social and civic actions. As a primary outcome of this

cluster, students will be able to construct and test arguments based on the validity of data and to understand the role probability plays in interpreting such arguments.

Students should have full access to calculators and computer software to assist in their study of these critical proficiencies and to ensure an efficient and thorough analysis of the data sets in this cluster.

Students will be able to:

1. Create and test an argument using displays or tables, given a set of statistics.

   Example: Given a newspaper account of a city council decision with relevant data, students can debate the correctness of the action taken.

2. Compare displays and interpretations with an eye to possible misuse of statistics.

   Example: Check graphs and compare with respect to the use of scale, adequate samples, and bias in data.

IV. Measurement and Geometry

Students should understand shapes and their properties and move from concrete to abstract understanding by using physical models, two- and three-dimensional figures, real-world objects, and computer simulations.

Students will be able to:

1. Use coordinate geometry to represent relationships.

   Example: inequalities

2. Select and use geometric models to represent and solve problems.

   Examples: congruency; similarity; relationships, e.g., ratios

3. Estimate and/or find perimeters, areas, and volumes of shapes.

   Example: Given a grid superimposed on an irregular region, estimate the area of the region.

V. Fundamentals of Algebra

Students will demonstrate manipulative facility and conceptual understanding of algebraic principles used in problem-solving situations. Students should be able to use mathematical models, real data, graphs, tables, expressions, equations, and technological applications.

Students will be able to:

1. Solve linear inequalities.

   Example: $2x > 3$

2. Solve a system of linear equations in two variables.

   Example: $x + y = 4$
   $x - y = 2$

3. Apply algebraic methods to solve a variety of problems.

   Example: growth and decay

NEW JERSEY STATE DEPARTMENT OF EDUCATION
DIVISION OF GENERAL ACADEMIC EDUCATION

CORE COURSE PROFICIENCIES STAFF

Carol N. Scelza ..... Manager, Bureau of Curriculum and Technology

Paul Cohen ..... Core Course Proficiencies Coordinator

Michael Ryan ..... Mathematics Panel Facilitator

Bruce Marganoff ..... Science Panel Facilitator

Deborah Cook ..... Science Proficiencies Coordinator

Julia Stapleton ..... Administrative Facilitator

Larry Wiley ..... Mathematics Consultant

Joseph Zahn ..... Mathematics Consultant

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